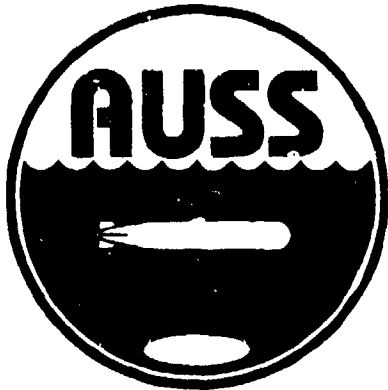
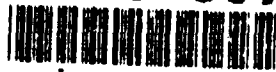


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Brownian Reber Search Theory for the Advanced Unmanned Search System (AUSS)

D. Grace



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**Brownian Reber
Search Theory
for the Advanced Unmanned
Search System (AUSS)**

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ADMINISTRATIVE INFORMATION

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Further information on this subject is available in related reports that represent NRaD efforts through FY 1992. The bibliography is found at the end of this report.

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Sciences Department

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EXECUTIVE SUMMARY

OBJECTIVE

This report summarizes the basic and Brownian Reber search theories which have been used to model the performance of the Advanced Unmanned Search System. It is intended to serve as a reference to the system modeling which influenced the design of the system. It should also serve as a foundation for the design of optimal searches.

APPROACH

Reber's original theory has been modified to incorporate navigation and control features typical of autonomous vehicle systems; the natural modification involves modeling vehicle trajectories as fractional Brownian paths. The general structure of the Brownian Reber theory is identical to the structure of the basic Reber theory, so that the usual figures of merit, such as mean time to detection, are calculable in terms of system measurables, such as navigation error and lateral range function.

RESULTS

The basic and Brownian theories are described in some detail, and their similarities and differences are explicitly analyzed. The form of the expression for mean time to detection is provided. The important parameters in the Brownian theory are determined numerically for a significant range of system variables, and these results are presented in a series of figures.

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INTRODUCTION

The Advanced Unmanned Search System (AUSS) was developed by the Naval Command, Control and Ocean Surveillance Center (NCCOSC) to improve the Navy's ability to find and identify items lost or placed on the sea floor at depths as great as 20,000 feet. Items such as the Palomares H-Bomb, the U.S.S. *Scorpion*, the U.S.S. *Thresher*, Korean Airlines Flight 007, Air India Flight 182, and the cargo door of United Airlines Flight 811 are examples of equipment lost by the U.S. and other countries. Searching for these items proved difficult and highlighted a critical technology area: deep ocean search.

The first purpose of a search theory model is to quantify the performance of a search system in terms of the parameters which most directly affect that performance, and as with any complicated system, this quantification necessitates a number of important simplifications. The object of this report is to explicitly state these simplifications, derive a theory based on these simplifications, and provide a parameterized summary of the performance of the AUSS which may be used in its engineering and operation.

In the continuing engineering of AUSS, the acceptability of such simplifications is a subject of constant discussion, particularly when the optimal operation of the system is being considered. This report is intended to document refinements to Reber's theory (Reber, 1956), which led to the initial design and development of AUSS. These refinements are suggested by a better understanding of AUSS as it actually exists, as opposed to how it might have been generally designed, and while it is accepted that the theory which is the subject of this report may itself be refined in time, it is complete within the limits of the simplified search scenario outlined in the next section.

The next section of this report outlines the important parametric simplifications, the resulting theory, and its application to the quantification of system performance. This report then discusses the AUSS navigation/control model and an explicit derivation of the location probability expressions. The RMS path deviation and mean search path length are also derived. Reber's theory is discussed in some detail, so that it will be clear where it differs from the modified theory which is the subject of this report. These differences are discussed with the object of substantiating the need for a modified theory. The next section concentrates on determining the factors which influence the time it takes to complete the n th coverage of the nominal search area, which in turn enters into the mean-time-to-detection figure-of-merit. Explicit expressions for various search scenarios of interest to AUSS are derived and explored. The final section summarizes the influence of the various system parameters through a series of figures.

SEARCH THEORY PARAMETERS

SEARCH SCENARIO

For AUSS, the simplified search scenario is as follows: The search vehicle, or search-sensor platform, is assumed to be searching a rectangular area by following a sequence of parallel paths and employing a search sensor, such as a sonar or a camera, which periodically provides information about or "searches" rectangular subareas along the path. In this description, the overall performance of the search system is the result of its performance in two essentially separable activities, that is, how well the system follows the sequence of parallel paths (the "nominal" search pattern) and how well the search sensor searches the subareas to which it is periodically applied. How well the system follows its nominal paths is a result of the accuracy and precision of the navigation data it uses, as well as the control algorithms which actuate the vehicle's thrusters, rudders, elevators, and other parts; how well a search sensor searches an area is described by the conditional probability of an object of interest being detected by the sensor, given that it is known to be present in the sensor's field of view.

This simplified search scenario is schematically depicted in figure 1. The search vehicle attempts to follow the nominal paths, of length L , in the directions indicated by the arrows superimposed on the paths; and at regular intervals separated by a distance δ along the nominal paths, a search sensor is employed to provide information about a subarea assumed to be centered on the position of the vehicle. These subareas have a width W (across track, or perpendicular to the nominal path) and a length δ (along track, or parallel to the nominal path), the dimension W being related to the range of the search sensor. The nominal paths are separated by a distance d , so that the nominal search area has width and length $H = Md$, $L = N\delta$, respectively, that is, there are M such nominal paths and N subareas along each nominal path.

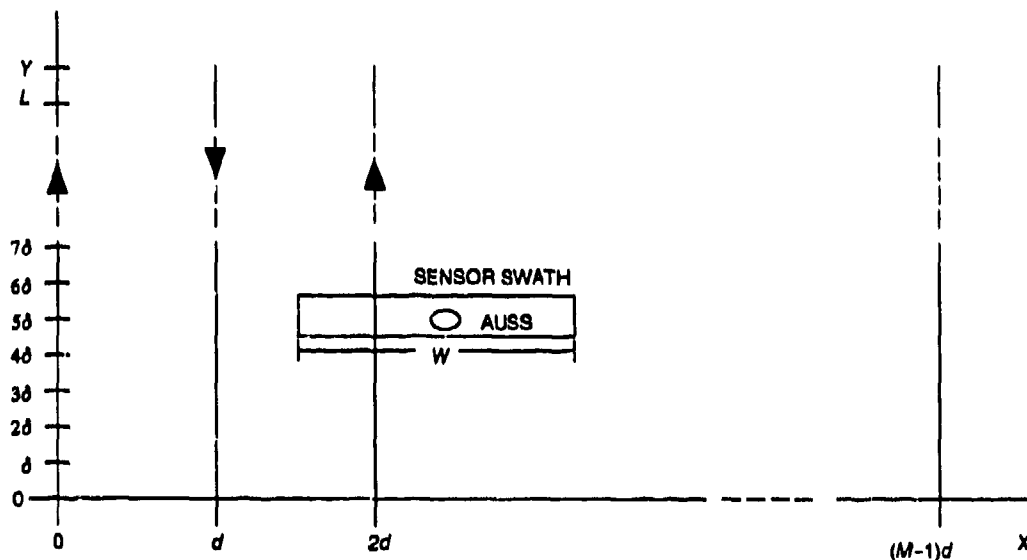


Figure 1. Nominal search scenario.

NAVIGATION ERROR

The model assumed to describe the vehicle's success in following the nominal search pattern is outlined in more detail below, but in short it states that the probability that the perpendicular distance of the vehicle from the path will change from one search interval to the next by an amount ϵ is given by

$$p(\epsilon)d\epsilon = \frac{1}{\sqrt{2\pi}\sigma^2(\delta)} \exp\left(-\frac{\epsilon^2}{2\sigma^2(\delta)}\right)d\epsilon$$

Note that the second moment of this distribution, σ , is a function of the distance between search swaths, δ , along the nominal path, although in the following expression, this dependence will usually be suppressed. In any event, this second moment is the so-called navigation error of the system. Most importantly, this model results in the square root of the mean perpendicular deviation from the nominal path (the RMS deviation) being an increasing function of distance along the path, with the specific form

$$\bar{\sigma}(y, \delta) = \left(\frac{y}{\delta}\right)^{1/2} \sigma$$

where y is the distance along a nominal path ($0 < y < L$). Note that the parameters

$$\{\bar{\sigma}, \delta, y\}$$

are measureables, so that this expression gives the search theory parameter σ as a function of the demonstrated capabilities of an actual system.

LATERAL RANGE FUNCTION

In general, the probability that a search sensor will detect an object of interest when it is known to be in the field of view of the sensor is a function of the object, its location in the field of view, the nature of the environment in which the sensor is being employed, and the performance of the operator or algorithm making use of the sensor information. The first simplification is to restrict the field of view of the sensor by indicating its effective range, which in the present situation is a perpendicular distance (the lateral range) from the sensor beyond which it has no chance of detecting an object of interest. For AUSS, the search swath is assumed to be symmetrical about the position of the sensor, so that the area searched by the sensor in one application has width W and length δ , as above, where the sensor has lateral range $W/2$. The second simplification lumps the remaining factors into a single number, a conditional probability β , which describes the probability that a given object in a given environment will be detected by a given operator or algorithm provided that it is within the field of view of the sensor. This is the so-called "cookie-cutter" lateral range function, and though quite crude it is a useful rendition of the actual sensor performance. Later reports will consider the effect on overall search performance of allowing β to be a true function of lateral range, as in $\beta = \beta(X)$, where X is the lateral range of the object of interest from the sensor.

LOCATION PROBABILITY

The probability that an object of interest will be detected after one attempted search of the nominal search area by following the sequence of parallel paths is the location probability, ϕ . There is a canonical form for ϕ which can be developed as follows. Suppose that the search vehicle follows the nominal search pattern with no error and detects objects in the field of view of its sensor with probability β , as above, and suppose further that the object of interest is certain to be located at least somewhere within the nominal search area. Since the search proceeds with no navigation or control error, it effectively partitions the nominal search area into NM subareas of size $W\delta$. Now, the probability that the object of interest is located within any one of these subareas is the ratio of the magnitude of the subarea to the magnitude of the whole search area, namely, $W\delta/HL$, so the probability of not detecting the object on one nominal coverage of the search area is

$$\left(1 - \frac{\beta W \delta}{HL}\right)^{NM}$$

Since the location probability is the probability that the object will be detected after one nominal coverage

$$\phi = 1 - \left(1 - \frac{\beta W \delta}{HL}\right)^{NM} = 1 - \left[\left(1 - \frac{\beta W}{HN}\right)^N\right]^M$$

or

$$\phi = 1 - \exp\left(-\frac{\beta WM}{H}\right) = 1 - \exp\left(-\frac{\beta W}{d}\right)$$

since $\delta = L/N$ and $d = H/M$. It is shown in the next section of this report that within certain reasonable limits on the relative values of the parameters discussed thus far, the location probability for a system which does not search with perfect navigation and control is well-approximated by

$$\phi_o = 1 - \exp\left(-\frac{\beta W}{d} \lambda_o\right)$$

where

$$\lambda_o = -\frac{\delta}{\beta WL} \sum_{k=1}^N \int_{-\frac{H}{2}}^{\frac{H}{2}} \ln \left| 1 - \frac{\beta}{\sqrt{2^k \pi} \sigma} \int_{-\frac{w}{2}}^{\frac{w}{2}} \exp\left(-\frac{x^2}{2^k \sigma^2}\right) dx \right| dz$$

The forms of the location probability and the other parameters are dictated by the desire to be consistent with Reber's theory. The parameter λ_o must be determined numerically, but in practice it is a convenient measure of the effect of imperfect navigation on overall search performance as quantified by the location probability ϕ .

MEAN TIME TO DETECTION

A real search is made up of a number of activities whose execution contributes to the overall time a search is said to require; these activities include those things which may be considered to directly contribute to the probability of finding an object of interest as well as those things which are more logistic in nature. In the former class is the search activity itself, that is, the use of the search sensor to provide information, while in the latter category are such things as transit time to the search area and system refurbishment time, in which it is assumed that the system has a limited deployment time which is less than the time required to conduct a single search of the nominal search area. Similarly, usually the case that search first proceeds with a sensor which views large areas rather quickly, such as a sonar, but which cannot distinguish between the object of interest and a set of other objects with similar signatures (false targets). In this case, a contact evaluation sensor is employed to distinguish between true and false contacts. The use of this second sensor is known as contact evaluation, and the time spent on contact evaluation depends upon the density of false targets in the nominal search area as well as the time it takes to evaluate a given contact, whether the contacts are evaluated as they occur or in groups at some later time.

It is important to note that it is assumed that more than one nominal coverage of the search area will be attempted before an object of interest is located. Denote by $T(n)$ the time spent on all activities during the n th attempted coverage of the nominal search area. Observe that in terms of the location probability ϕ , the probability that the object of interest will not be located until the n th attempted coverage is given by

$$P(n) = (1 - \phi)^{n-1}\phi$$

so the mean time spent searching for an object of interest is

$$T = \sum_{n=1}^{\infty} T(n)P(n) = \sum_{n=1}^{\infty} T(n)(1 - \phi)^{n-1}\phi$$

This mean time to detection is a useful figure of merit through which the effects may be investigated of the various system parameters on search system performance.

BROWNIAN REBER SEARCH THEORY

NAVIGATION BY DIRECTED RANDOM WALK

Consider a nominal search scenario as outlined above and represented schematically in figure 1. The AUSS vehicle attempts to follow the "square-wave" nominal search pattern by using a Doppler velocity sensor whose integrated output provides estimates of net changes in position in the search area; this position information is used by Type 1 control loop algorithms which attempt to keep the vehicle on the nominal path at every point. The net result of integrating a sensor's output (the Doppler velocity estimates) and using a line-following control algorithm based on integrated sensor output is modeled here by assuming that the probability of the perpendicular distance of the vehicle from the nominal path, evaluated at regular intervals along the nominal path, changing by an amount ϵ from one interval to the next, is

$$p(\epsilon)d\epsilon = \frac{1}{\sqrt{2\pi}\sigma(\delta)} \exp\left(-\frac{\epsilon^2}{2\sigma^2(\delta)}\right)d\epsilon$$

where δ is the separation of intervals along the nominal path.

In terms of figure 1, in which a position along the nominal path is described by the y-coordinate and a perpendicular position relative to the nominal path $x = x_o$ is described by $x - x_o$

$$p(\epsilon)d\epsilon = p(x_n|x_{n-1})dx_n = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x_n - x_{n-1})^2}{2\sigma^2}\right]dx_n$$

Here the index n runs from $n = 1$ to $n = N$, where $N = L/\delta$, L being the length of the nominal paths, which are successively described by $x_o = qd$, the index q running from $q = 0$ to $q = M - 1$, where $M = H/d$, H being the width of the search area and d the separation of the nominal paths.

The probability of the vehicle following a path described by a specific sequence of perpendicular positions $\{x_1, x_2, \dots, x_N\}$ at the intervals $\{\delta, 2\delta, \dots, N\delta\}$ along the nominal path is then

$$p(x_1|x_o)dx_1 \cdot p(x_2|x_1)dx_2 \cdot \dots \cdot p(x_N|x_{N-1})dx_N$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^N \exp\left[-\sum_{n=1}^N \frac{(x_n - x_{n-1})^2}{2\sigma^2}\right]dx_1dx_2\dots dx_N \quad (1)$$

Equation 1 is the fundamental directed random walk assumption which determines the navigation/control performance of the system.

LOCATION PROBABILITY

Now consider, as above, a search sensor with a cookie-cutter lateral range function with swath width W and detection probability β . If the object of interest is located at (x_i, y_i) within the nominal search area, then $y_k \leq y_i \leq y_{k+1}$ for some $1 \leq k \leq N$. The probability of detecting the object while searching the nominal path $x = x_o$ is then

$$P_k(x_o) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^N \int_{-\infty}^{\infty} \dots \int_{x_i - \frac{W}{2}}^{x_i + \frac{W}{2}} \beta \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \exp\left[-\frac{\sum_{n=1}^N (x_n - x_{n-1})^2}{2\sigma^2}\right] dx_1 \dots dx_k \dots dx_N$$

since the object is detected with probability β when $x_i - W/2 \leq x \leq x_i + W/2$ and $y = y_i$, and zero otherwise. Straightforward integration with changes of order and variable gives

$$P_k(x_o) = \frac{\beta}{\sqrt{2^k \pi} \sigma} \int_{x_i - x_o - \frac{W}{2}}^{x_i - x_o + \frac{W}{2}} \exp\left(-\frac{x^2}{2^k \sigma^2}\right) dx \quad (2)$$

Finally, observe that the probability of failing to detect the object while searching the nominal path $x = x_o$ is $1 - P_k(x_o)$, so the probability of failing to find the object on one nominal coverage of the search area is

$$[1 - P_k(x_o = 0)][1 - P_k(x_o = d)] \dots [1 - P_k(x_o = (M-1)d)] \equiv 1 - \phi(x_i, y_i) \quad (3)$$

Equations 2 and 3 define the location probability, $\phi(x_i, y_i)$; a more useful quantity is the average of $\phi(x_i, y_i)$ over all possible locations of the object within the nominal search area. Provided that the magnitude of the area of the paths along the edges of the search area is small relative to the magnitude of the total area, averaging on x_i can be performed over the smaller range

$$x' - d/2 \leq x_i \leq x' + d/2$$

for some x' well-removed from the edges of the nominal search area. This gives the average location probability

$$\phi \equiv \frac{1}{dL} \int_0^L \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \phi(x_i, y_i) dx_i dy_i \quad (4)$$

or, more explicitly

$$\phi \equiv \frac{1}{dL} \int_0^L \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \left\{ 1 - \prod_{q=0}^{M-1} [1 - P_k(x_o = qd)] \right\} dx_i dy_i$$

Now define

$$\lambda \equiv -\frac{d}{\beta W} \ln \left\{ \frac{1}{dL} \int_0^L \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \prod_{q=0}^{M-1} [1 - P_k(x_0 = qd)] dx, dy_i \right\} \quad (5)$$

then ϕ has the familiar form

$$\phi = 1 - \exp\left(-\frac{\beta W}{d} \lambda\right) = 1 - \exp(-m\lambda)$$

with $m = \beta W/d$, the nominal path density.

It is possible to considerably simplify Eq. 5 for λ ; first write

$$\int_0^L \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \prod_{q=0}^{M-1} [1 - P_k(x_0 = qd)] dx, dy_i = \int_0^L \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \exp \left\{ \sum_{q=0}^{M-1} [1 - P_k(x_0 = qd)] \right\} dx, dy_i$$

The argument of the exponential in the latter integrand varies slowly over the range of integration for a considerable number of the likely values of the parameters of interest here; it is shown later in this report over exactly what range of these parameters this argument may be replaced by its average over the nominal search area. Replacing the argument by its average gives

$$\int_0^L \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \prod_{q=0}^{M-1} [1 - P_k(x_0 = qd)] dx, dy_i \approx dL \exp \left\{ \frac{1}{dL} \int_0^L \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \sum_{q=0}^{M-1} [1 - P_k(x_0 = qd)] dx, dy_i \right\}$$

leading to an approximation of λ

$$\lambda_o \equiv -\frac{d}{\beta WL} \int_0^L \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \sum_{q=0}^{M-1} \ln[1 - P_k(x_0 = qd)] dx, dy_i$$

and an approximation of ϕ

$$\phi_o = 1 - \exp(-m\lambda_o)$$

It remains only to simplify the expression for λ_o . Notice first that the inner integral over x_i is constant for $y_k \leq y_i \leq y_{k+1}$, so that the integral over y_i is actually a sum:

$$\lambda_o \equiv -\frac{d}{\beta WL} \cdot \delta \sum_{k=1}^N \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \sum_{q=0}^{M-1} \ln[1 - P_k(x_0 = qd)] dx_i$$

each term in the sum being multiplied by

$$\int_{y_k}^{y_{k+1}} dy_i = y_{k+1} - y_k = \delta$$

Using the explicit form for $P_k(x_o)$ and interchanging the sum on q with the integration over x gives

$$\lambda_o = -\frac{\delta}{\beta WL} \sum_{k=1}^N \sum_{q=0}^{M-1} \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \ln \left| 1 - \frac{\beta}{\sqrt{2^k \pi} \sigma} \int_{x_i - qd - \frac{w}{2}}^{x_i - qd + \frac{w}{2}} \exp\left(-\frac{x^2}{2^k \sigma^2}\right) dx \right| dx_i$$

Making the change of variable $z = x_i - qd$ for each term in the sum over q gives

$$\lambda_o = -\frac{\delta}{\beta WL} \sum_{k=1}^N \sum_{q=0}^{M-1} \int_{x' - (n + \frac{1}{2})d}^{x' - (n - \frac{1}{2})d} \ln \left| 1 - \frac{\beta}{\sqrt{2^k \pi} \sigma} \int_{z - \frac{w}{2}}^{z + \frac{w}{2}} \exp\left(-\frac{x^2}{2^k \sigma^2}\right) dx \right| dz$$

$$= -\frac{\delta}{\beta WL} \sum_{k=1}^N \int_{x' - (M - \frac{1}{2})d}^{x' - \frac{d}{2}} \ln \left| 1 - \frac{\beta}{\sqrt{2^k \pi} \sigma} \int_{z - \frac{w}{2}}^{z + \frac{w}{2}} \exp\left(-\frac{x^2}{2^k \sigma^2}\right) dx \right| dz$$

Since $x = (H - d)/2$ is the x -coordinate most distant from the edges of the search area, using $x' = (H - d)/2$ in the last expression gives the form discussed above:

$$\lambda_o = -\frac{\delta}{\beta WL} \sum_{k=1}^N \int_{-\frac{H}{2}}^{\frac{H}{2}} \ln \left| 1 - \frac{\beta}{\sqrt{2^k \pi} \sigma} \int_{z - \frac{w}{2}}^{z + \frac{w}{2}} \exp\left(-\frac{x^2}{2^k \sigma^2}\right) dx \right| dz \quad (6)$$

RMS PATH DEVIATION AND MEAN PATH LENGTH

The square root of the mean of the square of the perpendicular deviation of the actual vehicle path from the nominal path is a function of the distance along the nominal path. With these successive distances given by $y_k = k\delta$, as above, where the index k runs from $k = 1$ to $k = N$, the RMS path deviation is found from

$$\sigma^2(y_k) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^k \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} (x_k - x_o)^2 \exp\left[-\frac{\sum_{n=1}^k (x_n - x_{n-1})^2}{2\sigma^2}\right] dx_1 \dots dx_k$$

Set

$$I = \left(\frac{1}{\sqrt{2\pi}\sigma} \right) \int_{-\infty}^{\infty} (x_k - x_o)^2 \exp \left[-\frac{(x_k - x_{k-1})^2}{2\sigma^2} \right] dx_k$$

Explicit evaluation trivially gives

$$I = \sigma^2 + (x_{k-1} - x_o)^2$$

Changing the order of integration

$$\begin{aligned} \sigma^2(y_k) &= \left(\frac{1}{\sqrt{2\pi}\sigma} \right)^{k-1} \int_{-\infty}^{\infty} \cdot \int_{-\infty}^{\infty} [\sigma^2 + (x_{k-1} - x_o)^2] \exp \left[-\frac{\sum_{n=1}^{k-1} (x_n - x_{n-1})^2}{2\sigma^2} \right] dx_{k-1} \cdot dx_1 \\ &= \sigma^2 + \left(\frac{1}{\sqrt{2\pi}\sigma} \right)^{k-1} \int_{-\infty}^{\infty} \cdot \int_{-\infty}^{\infty} (x_{k-1} - x_o)^2 \exp \left[-\frac{\sum_{n=1}^{k-1} (x_n - x_{n-1})^2}{2\sigma^2} \right] dx_{k-1} \cdot dx_1 \\ &= k\sigma^2 = \frac{y_k}{\delta} \sigma \end{aligned}$$

or, more loosely

$$\sigma^2(y) = \left(\frac{y}{\delta} \right)^{\frac{1}{2}} \sigma \quad (7)$$

Note that it is implicit here that each nominal path is begun correctly.

In the parametric studies of this report, the RMS deviation will be summarily represented by the constant

$$\sigma = \sigma \left(y = \frac{L}{2} \right) = \left(\frac{N}{2} \right)^{\frac{1}{2}} \sigma$$

It is similarly straightforward to find the mean path length, with

$$L = \left(\frac{1}{\sqrt{2\pi}\sigma} \right)^N \int_{-\infty}^{\infty} \cdot \int_{-\infty}^{\infty} \sum_{m=1}^N \sqrt{(x_m - x_{m-1})^2 + \delta^2} \exp \left[-\frac{\sum_{n=1}^N (x_n - x_{n-1})^2}{2\sigma^2} \right] dx_1 \cdot dx_N$$

giving the mean length of a single nominal path, so that the total mean path length for one nominal coverage is the sum of M such quantities. It is easy to see that

$$\begin{aligned} L &= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \sum_{m=1}^N \sqrt{(x_m - x_{m-1})^2 + \delta^2} \exp \left[-\frac{(x_m - x_{m-1})^2}{2\sigma^2} \right] dx_m \\ &= NI_o = \frac{L}{\delta} I_o \end{aligned}$$

where

$$I_o = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \sqrt{x^2 + \delta^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \quad (8)$$

It is possible to find an asymptotic form for I_o , which is reasonably accurate for $\delta \ll L$ and $\sigma < \delta$, using the method of steepest descents (Jeffreys and Jeffreys, 1956):

$$I_o = \frac{\delta}{\sqrt{1 - \left(\frac{\sigma}{\delta}\right)^2}}, \text{ for } \sigma < \delta \text{ and } \delta \ll L$$

and

$$I_o \approx \sigma \sqrt{-\frac{2}{1 - \left(\frac{\delta}{\sigma}\right)^2}} \exp\left\{-\frac{1}{2} \left|1 - \left(\frac{\delta}{\sigma}\right)^2\right|\right\}, \text{ for } \sigma > \delta \text{ and } \delta \ll L$$

The approximations of I_o can be used to further illustrate the differences between Reber's basic theory and a Brownian Reber theory. In the Brownian theory, the vehicle paths are well-known fractals (Mandelbrot, 1982), and the fractal dimension of these paths serve to reveal these fundamental differences. The expression I_o is the length of a segment of the vehicle's path, so the finite, D -dimensional measure of an entire path of $N = L/\delta$ such segments is

$$L_D = \lim_{\delta \rightarrow 0} \frac{L}{\delta} \mu^D(\delta) = \lim_{\delta \rightarrow 0} \frac{L}{\delta} I_o^D(\delta)$$

where the D -dimensional measure of each segment is $\mu^D(\delta) = I_o^D(\delta)$.

The two approximations of I_o given above distinguish two fundamentally different types of path. For $\sigma < \delta$

$$\begin{aligned} I_o &= \frac{\delta}{\left[1 - \left(\frac{\sigma}{\delta}\right)^2\right]^{\frac{1}{2}}} \\ \Rightarrow \frac{L}{\delta} I_o^D &\approx L \delta^{D-1} \left[1 - \left(\frac{\sigma}{\delta}\right)^2\right]^{-\frac{D}{2}} \\ \Rightarrow L_D &= \lim_{\delta \rightarrow 0} L \delta^{D-1} \left[1 - \left(\frac{\sigma}{\delta}\right)^2\right]^{-\frac{D}{2}} \end{aligned}$$

Now

$$\sigma \ll \delta \Rightarrow \left[1 - \left(\frac{\sigma}{\delta}\right)^2\right]^{-\frac{D}{2}} \approx 1 + \frac{D}{2} \left(\frac{\sigma}{\delta}\right)^2$$

so if $D > 1$

$$L_D = \lim_{\delta \rightarrow 0} L \delta^{D-1} \frac{D}{2} \left(\frac{\sigma}{\delta}\right)^2$$

By presumption L_D is finite, so $D > 1 \Rightarrow \sigma \approx \delta^{\frac{3-D}{2}}$, which together with $\sigma \ll \delta$ requires $D \leq 1$, a contradiction. Hence, for $\sigma \ll \delta$ as $\delta \rightarrow 0$, the path has dimension $D = 1$ and the limiting path length is L .

On the other hand, for $\sigma > \delta$

$$I_0 = \sigma \left[\frac{2}{1 - \left(\frac{\delta}{\sigma}\right)^2} \right]^{\frac{1}{2}} \exp \left\{ -\frac{1}{2} \left[1 - \left(\frac{\delta}{\sigma}\right)^2 \right] \right\}$$

and a similar limiting analysis shows that

$$L_D = \lim_{\delta \rightarrow 0} \frac{L}{\delta} \sigma^D \left(\frac{2}{e}\right)^{\frac{D}{2}}$$

Again the presumed finitude of L_D requires $\sigma \approx \delta^{\frac{1}{D}}$ or $\sigma = \eta \delta^{\frac{1}{D}}$ for some $\eta \geq 1$, which is the description of a fractional Brownian path (Mandelbrot and van Ness, 1968) of dimension $1 \leq D \leq 2$. The parameters η and D are determined from

$$\log \sigma = \log \eta + \frac{1}{D} \log \delta$$

using linear regression on the measurables σ and δ ; note that the fractal path length is then

$$L_D = \eta^D \left(\frac{2}{e}\right)^{\frac{D}{2}} L$$

The difference between the two types of path, namely, those for which $\sigma < \delta$ and $\sigma > \delta$, is essentially the difference between those paths whose pointwise variance vanishes faster than the decreasing scale at which it is evaluated and those whose pointwise variance vanishes more slowly. The latter paths are the model chosen here, since choosing the former implies that the vehicle's path evaluated at discrete times is smoother than in practice it appears to be (Walton, 1992a, 1992b).

Two fundamental differences between the basic Reber theory and a Brownian Reber theory are explicit now, the first already identified above: the RMS path deviation is an

increasing function of distance along the nominal path, which is characteristic of systems which employ Type 1 control loops. In the basic theory, the navigation error and the RMS path deviation are identical and constant along the path, as indicated in the summary discussion of the basic theory given below. The basic Reber path is consequently two-dimensional, while the above argument shows that paths in a Brownian theory have dimension $1 \leq D \leq 2$, where D is determined experimentally. This second fundamental difference between the basic and Brownian theories, as well as the need to model navigation using Type 1 control loops shown by the first difference, led to the Brownian theory which is the subject of this report.

Figure 2 shows a nominal Reber path and three Brownian Reber paths for three fractal dimensions $1 < D < 2$; all four paths are of equal probability. Figure 3 shows the path multiplier

$$a = \frac{L}{L_0} = \frac{l_0}{\delta} \approx \eta \delta^{\frac{1}{D}-1} \left(\frac{2}{e} \right)^{\frac{1}{2}}$$

as $\delta \rightarrow 0$ with $\eta = 1$.

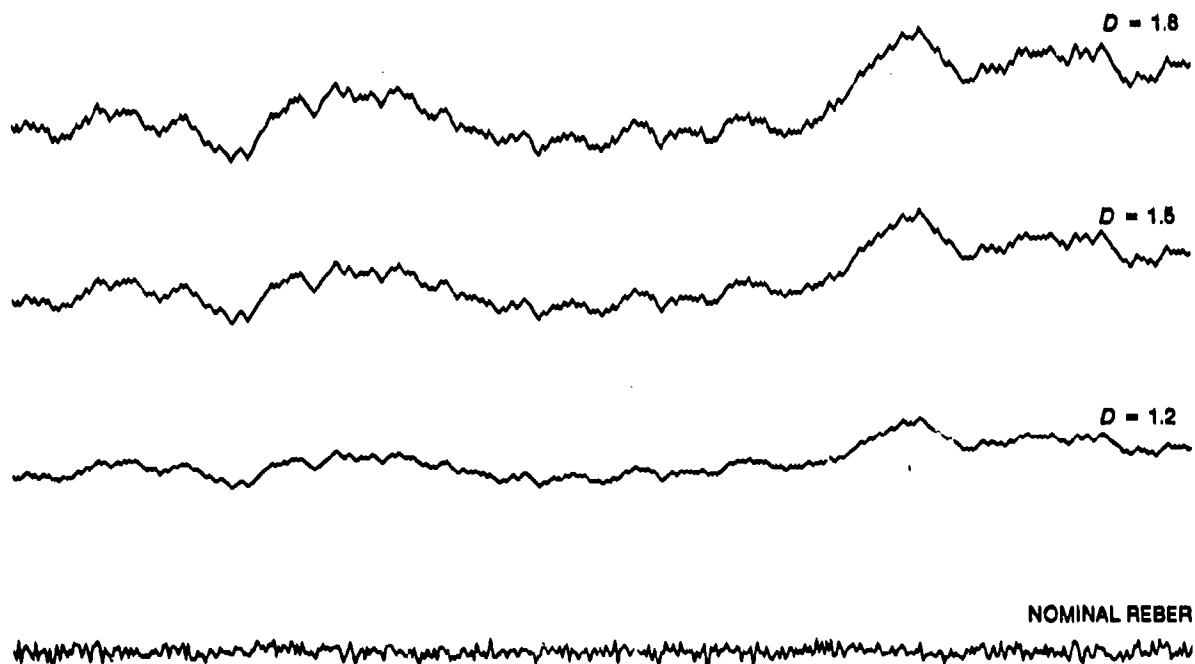


Figure 2. Equiprobable nominal Reber and Reber-Brownian paths of fractal dimension D .

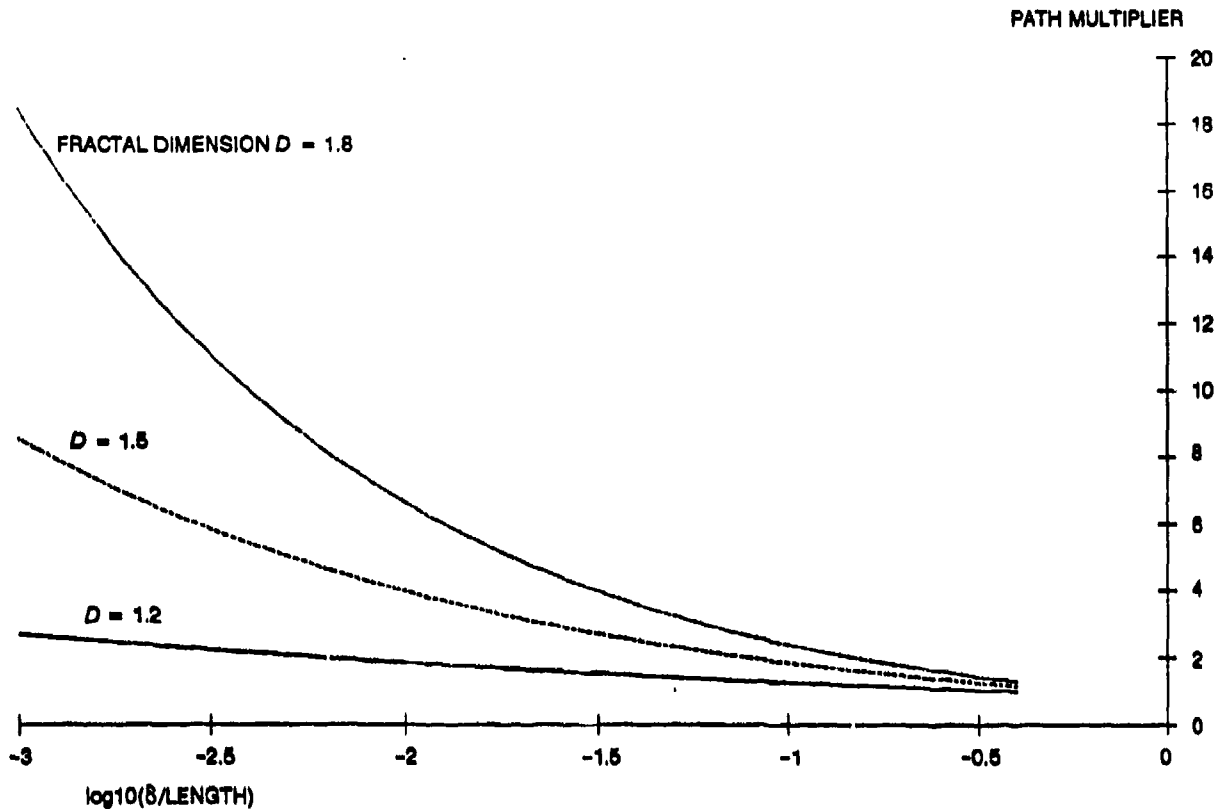


Figure 3. Path multiplier as a function of σ_0 .

REBER'S THEORY IN OUTLINE

To a great extent, Reber's theory of search for narrow path locators underlies all of the above, and since this report is intended to document a significant modification to this basic theory, it is appropriate here to present it in outline. The main points of divergence between the modified and basic theories will be noted as they are developed.

The fundamental difference, from which all others follow, lies in the form which the navigation error is assumed to take. For the basic theory, the probability that the search vehicle is within ϵ of the nominal path is the same as the probability in the modified theory of a change of magnitude ϵ in the x -coordinates of succeeding positions. As the vehicle attempts to follow the nominal path $x = x_0$, for any y -coordinate along the path, the x -coordinate has probability

$$p(x) = \frac{\beta}{\sqrt{2\pi}\bar{\sigma}} \exp\left[-\frac{(x-x_0)^2}{2\bar{\sigma}^2}\right] dx \quad (9)$$

where the navigation error is now quantified by the parameter $\bar{\sigma}$.

This is to be compared with Eq. 1. Note that in the simplified search scenario discussed above, navigation error for the modified theory is a function of the along-track

evaluation interval δ . In the basic theory, the navigation error is independent of any such parameter, and as a consequence, the RMS path deviation is identical to this navigation error. Figure 2 shows four typical paths, of identical relative probability, for these differing basic and Brownian navigation assumptions. It is the demonstrated similarity of the path for the modified theory to actual AUSS paths and the assumed consequences of the use of a Doppler velocity sensor whose integrated output provides estimates of relative position for line-following Type 1 control loop algorithms, which led to the development of the Brownian Reber Theory.

For a cookie-cutter lateral range function with a conditional probability β , with the identical simplified search scenario discussed above, the probability of detecting the object while searching the nominal path $x = x_0$ is

$$P_k(x_0) = \frac{\beta}{\sqrt{2^k \pi} \sigma} \int_{x_1 - x_0 - \frac{W}{2}}^{x_1 - x_0 + \frac{W}{2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \quad (10)$$

which is to be compared with Eq. 2. This leads to a location probability in the equation

$$[1 - P(x_0 = 0)][1 - P(x_0 = d)] \dots [1 - P(x_0 = (M-1)d)] = 1 - \phi(x_t, y_t) \quad (11)$$

which may be compared with Eq. 3. This is then averaged over x -coordinates of object locations in the range $x' - d/2 \leq x_t \leq x' + d/2$ to give

$$\phi = \frac{1}{d} \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \left\{ 1 - \prod_{q=0}^{M-1} [1 - P(x_0 = qd)] \right\} dx_t$$

Notice that the modified theory also averages the location probability over the y -coordinate of object locations; in the basic theory, the y -coordinate plays no role.

Now defining

$$\gamma = -\frac{d}{\beta W} \ln \left\{ \frac{1}{d} \int_{x' - \frac{d}{2}}^{x' + \frac{d}{2}} \prod_{q=0}^{M-1} [1 - P(x_0 = qd)] dx_t \right\} \quad (12)$$

the location probability takes the canonical form

$$\phi = 1 - \exp\left(-\frac{\beta W}{d} \gamma\right)$$

Equation 12 may be compared with Eq. 5. The simplification of the expression for γ proceeds in essentially the same fashion as that of λ leading to the approximation γ_0 :

$$\gamma_o = -\frac{1}{\beta W} \int_{-\frac{H}{2}}^{\frac{H}{2}} \ln \left| 1 - \frac{\beta}{\sqrt{2^k \pi} \sigma} \int_{z-\frac{W}{2}}^{z+\frac{W}{2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \right| dz \quad (13)$$

This last expression is to be compared with Eq. 6.

As indicated above, the RMS path deviation for the basic theory is identical to the navigation error, σ . Since the navigation error is not a function of the along-track evaluation interval, δ , a mean path length is not calculated since it leads to unbounded estimates of mean path length in the basic theory.

In any event, once a location probability is available, the mean time to detection is calculated in precisely the same fashion. The utility of Reber's theory is due in part to the parametric summarization of the dependence of the location probability on navigation error, path separation, swath width, and lateral range function, presented in a series of figures which accompany the documentation of the theory (Reber, 1956). A similar summarization is included in a later section of this report.

FIGURE OF MERIT

TOTAL TIME FOR THE n th COVERAGE

As discussed earlier, a real search consists of a number of activities which contribute to the total time a search is said to take, including the time spent transitting to the search area, time spent actually employing search sensors to provide information, time spent evaluating contacts, and time spent on support, logistic, and general housekeeping activities, during which the search is said to be in abeyance. For the moment, the time spent initializing the search will be taken for granted and will not appear in any expressions for mean time to detection. Since it is assumed that more than one coverage of the nominal search area will probably be needed before an object is detected, denote by $T(n)$ the total time spent on the n th coverage; then summarizing these remarks

$$T(n) = T_s(n) + T_c(n) + T_A(n)$$

where $T_s(n)$ is the time spent actually employing search sensors to provide information, or the search time, $T_c(n)$ is the time spent investigating contacts, or the contact evaluation time, and $T_A(n)$ is the time spent on all other activities, or the abeyance time. With ϕ the location probability, the mean time to detection, also discussed earlier is

$$T = \sum_{n=1}^{\infty} T(n) P(n) = \sum_{n=1}^{\infty} T(n) \phi (1 - \phi)^{n-1} = \phi \sum_{n=1}^{\infty} [T_s(n) + T_c(n) + T_A(n)] (1 - \phi)^{n-1}$$

SEARCH TIME

Assuming that the nominal search pattern consists as above of M parallel paths of length L and separated by a distance d and that the vehicle while actually searching moves with speed v , the time spent employing search sensors to provide information is proportional to the number of coverages, as in

$$T_s(n) = n \frac{ML}{v}$$

where L is the mean path length discussed earlier.

CONTACT EVALUATION TIME

It is assumed that the search proceeds first by using a sensor which has a large field of view (a broad area search sensor, such as a sonar) but which cannot distinguish between the object of interest and a set of objects which have similar signatures, i.e., false targets; these contacts are evaluated by using a separate sensor (such as a high-resolution CCD camera). The total time spent on contact evaluation is then a function of the number of false targets present in the nominal search area, quantified by the false target density, f , and the time spent investigating each contact, quantified by the contact evaluation time, τ . It is assumed here that each contact is immediately

evaluated and that this leads to its elimination as a potential contact, whether true or false. This assumption is known as immediate contact evaluation with elimination.

Since a false target is indistinguishable from the object of interest by the broad area search sensor, it has the same location probability, ϕ , as the object of interest, and since the nominal search area has magnitude HL , the number of false contacts generated on the first attempted coverage is taken to be $(fHL)\phi$. The number remaining to be detected on the second coverage is then

$$fHL - fHL\phi = fHL(1 - \phi)$$

which leads to $fHL(1 - \phi)\phi$ false contacts on the second attempted coverage. In general, then, the time spent evaluating contacts after n attempted coverages of the nominal search area is taken to be

$$T_C(n) = \sum_{r=1}^n fHL\phi(1 - \phi)^{r-1}\tau = fHL\tau[1 - (1 - \phi)^n]$$

ABEYANCE TIME

Since a typical nominal search area can be as large as 100 sq nmi, it must be assumed that the search vehicle will be deployed and recovered many times during a nominal coverage. In particular, for AUSS a certain amount of time per deployment is spent on descending to search depth, ascending from search depth, and maneuvering at search depth, maneuvering consisting at the very least of turning from one nominal path to the next. Likewise, each deployment is likely to involve a refurbishment of the vehicle (battery changeouts, for example), as well as other logistics. Suppose, for each deployment, that T_1 is the time spent for descent, T_2 the time spent for ascent, T_M the time spent maneuvering at depth, T_R the time spent for vehicle refurbishment, and T_L the time spent on other logistics, and assume that these quantities remain constant for each deployment. If N_D is the number of deployments in any one nominal coverage, assumed to remain constant for each coverage, the abeyance time is

$$T_A(n) = nN_D(T_1 + T_2 + T_M + T_R + T_L)$$

Note that all these quantities are specified by the system, with the exception of the maneuvering time and the number of deployments per nominal coverage. Assuming that the vehicle will follow true paths of length

$$L_i \leq L$$

before turning onto adjacent paths, there would be

$$\frac{ML}{L_i}$$

such turning maneuvers during one nominal coverage of the search area, and if T_i is the time spent on one turn

$$N_D T_M = \frac{ML}{L_s} T_i = \frac{ML}{L_s} T_i$$

the same scale factor multiplying lengths.

The number of deployments is related to the deployment time or endurance time of the vehicle, denoted T_D , typically set by batteries or other such factors. Explicitly

$$nN_D T_D = T_S(n) + nN_D(T_1 + T_2 + T_M) + T_C(n)$$

Using all of these expressions gives

$$T_A(n) = \left[\frac{T_S(n) + T_C(n)}{T_D - T_1 - T_2} \right] \left(T_1 + T_2 + T_R + T_L + \frac{nML}{L_S} T_i \right)$$

MEAN TIME TO DETECTION

Note first that

$$\sum_{n=1}^{\infty} T_S(n) \phi (1-\phi)^{n-1} = \sum_{n=1}^{\infty} n \frac{ML}{v} \phi (1-\phi)^{n-1} = \frac{ML}{v\phi}$$

and

$$\sum_{n=1}^{\infty} T_S(n) n \phi (1-\phi)^{n-1} = \frac{ML}{v\phi^2}$$

Likewise

$$\sum_{n=1}^{\infty} T_C(n) \phi (1-\phi)^{n-1} = \sum_{n=1}^{\infty} fHL\tau [1 - (1-\phi)^n] \phi (1-\phi)^{n-1} = \frac{fHL\tau}{2-\phi}$$

and

$$\sum_{n=1}^{\infty} T_C(n) n \phi (1-\phi)^{n-1} = fHL\tau \frac{(1-\phi)}{\phi(2-\phi)}$$

These give

$$T = \left(\frac{T_D + T_R + T_L}{T_D - T_1 - T_2} \right) \left(\frac{ML}{v\phi} + \frac{fHL\tau}{2-\phi} \right) + \frac{MLT_i}{L_S} \left[\frac{ML}{v\phi^2} + \left(\frac{fHL\tau}{2-\phi} \right) \left(\frac{1-\phi}{\phi} \right) \right]$$

PARAMETRIC STUDIES

SEARCH DESIGN BY MEAN TIME TO DETECTION

The purpose of these parametric studies is twofold, the larger purpose being to facilitate the design of a search by quantifying its effectiveness in terms of a figure of merit, the mean time to detection. Typically the target of interest and the nature of the terrain determine the lateral range function and swath width, with navigation error given for a specific system configuration. Hence, the twofold purpose of this study follows the nature of the design of a search, which consists of first choosing a path separation sufficient to ensure an acceptable location probability for a given lateral range function, swath width, and navigation error, and then evaluating the effectiveness of this search by the associated mean time to detection.

The figures presented below are intended to illustrate the dependence of λ , λ_o , location probability, and ultimately mean time to detection on such system parameters as swath width, navigation error, lateral range function, and path separation. The validity of the approximation λ_o is considered first, both in terms of its closeness to λ and to two associated location probabilities, ϕ and ϕ_d . This validity is important here because the approximation λ_o is used to determine the desired path separation, as outlined below. As stated above, the figure of merit of greatest interest is mean time to detection; the relevant quantity evaluated here is mean search time, given as a function of swath width, navigation error, location probability, and lateral range function.

VALIDITY OF THE APPROXIMATION λ_o

Figure 4 shows the parameter λ as a function of swath width, path separation, and navigation error for a typical cookie-cutter lateral range function $\beta = 0.9$; the RMS deviation σ was taken to be the RMS deviation at a point midway along the nominal path, as discussed earlier. Figure 5 gives $\frac{\lambda_o}{\lambda}$ for the same range of parameters. In both figures, the quantities evaluated are

$$\lambda = -\frac{d}{\beta W} \ln \left\{ \frac{\delta \sigma}{L d} \int_0^{\frac{d}{\sigma}} \sum_{k=1}^N \prod_{q=0}^{M-1} \left[1 - \frac{\beta}{\sqrt{\pi}} \int_{\frac{s + \frac{H - (1+2q)d - w}{\sigma} \frac{k}{2^{1+\frac{k}{2}}}}^{\frac{s + \frac{H - (1+2q)d + w}{\sigma} \frac{k}{2^{1+\frac{k}{2}}}} \exp(-x^2) dx \right] dz \right\} \quad (14)$$

$$\lambda_o = -\frac{\sigma \delta}{\beta W L} \int_{-\frac{H}{2\sigma}}^{\frac{H}{2\sigma}} \sum_{k=1}^N \ln \left[1 - \frac{\beta}{\sqrt{\pi}} \int_{\frac{s - \frac{W}{\sigma} \frac{k}{2^{\frac{k}{2}}}}^{\frac{s + \frac{W}{\sigma} \frac{k}{2^{\frac{k}{2}}}} \exp(-x^2) dx \right] dz \quad (15)$$

with H , L an order of magnitude greater than d , δ , respectively.

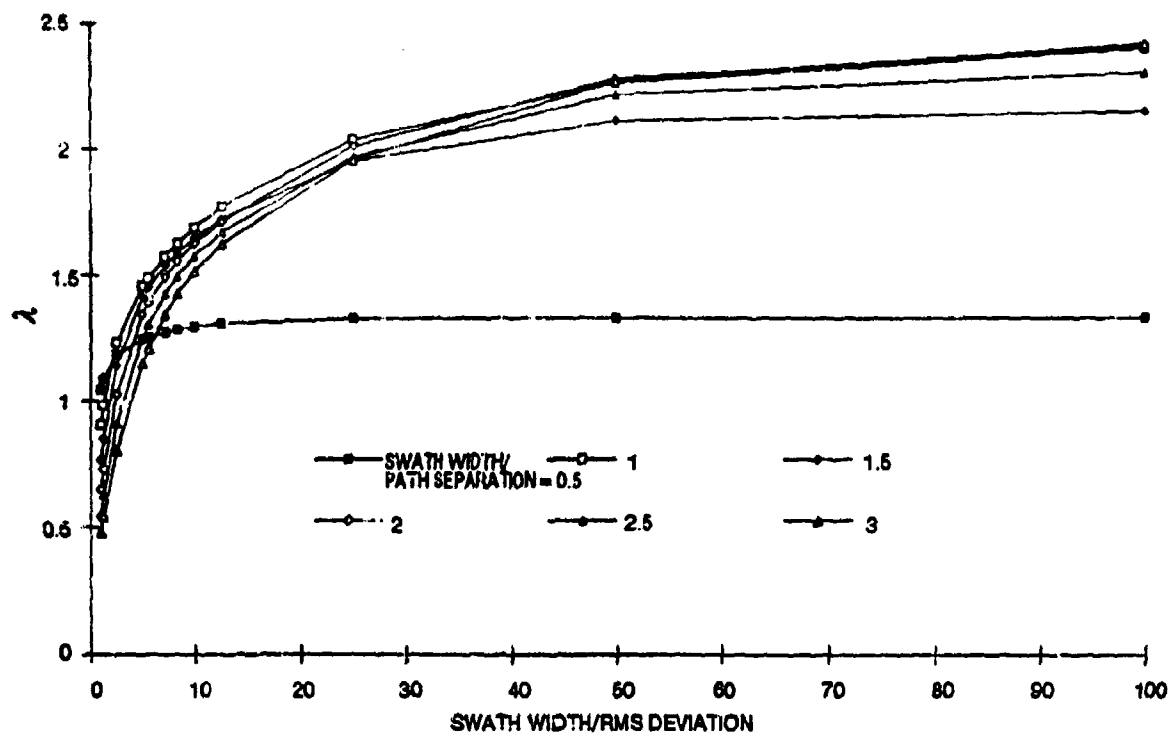


Figure 4. λ as a function of swath width for $\beta = 0.9$.

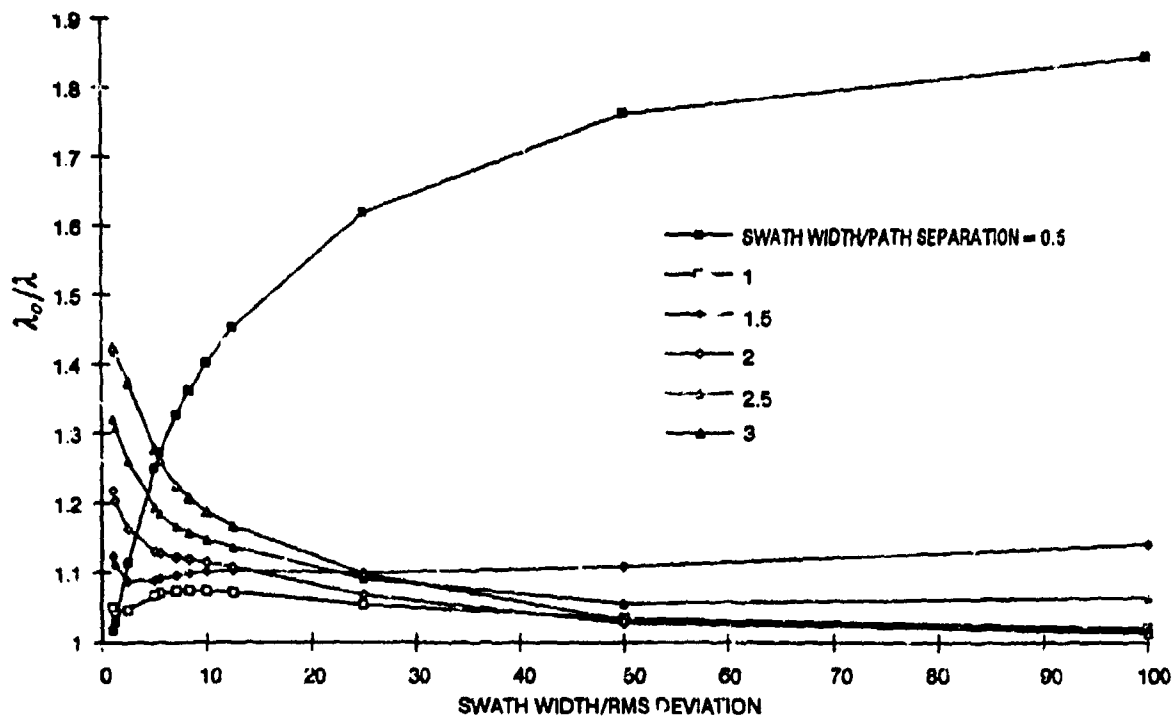


Figure 5. λ_0/λ as a function of swath width for $\beta = 0.9$.

Using Eq. 15 for λ_o and the definition $\phi_o = 1 - \exp\left(-\lambda_o \frac{b}{Wd}\right)$ gives an approximation for λ_o the path separation d required to ensure this location ϕ_o :

$$\frac{d}{\sigma} = -\frac{\frac{1}{2^2}}{N^2 \ln(1 - \phi_o)} \int_{-\frac{H}{2\sigma}}^{\frac{H}{2\sigma}} \sum_{k=1}^N \ln \left[1 - \frac{\beta}{\sqrt{\pi}} \int_{\frac{z - \frac{W}{\sigma}}{2^{\frac{1}{2}}}}^{\frac{z + \frac{W}{\sigma}}{2^{\frac{1}{2}}}} \exp(-x^2) dx \right] dz \quad (16)$$

The most important estimation of the validity of the approximation λ_o is given in figures 6a-d, which show the relative error between the probability ϕ and a derived location probability

$$\phi_d = 1 - \exp\left(-\lambda \frac{\beta}{Wd}\right)$$

in which the path separation d is determined from Eq. 16 and λ is then determined from Eq. 14. These figures suggest using λ_o to determine a path separation given ϕ , β , W , and σ is acceptable for a reasonable range of these parameters, where acceptability is taken to be

$$\frac{\|\phi_d - \phi\|}{\phi} \leq 0.05$$

PATH SEPARATION AND MEAN SEARCH TIME PARAMETRICALLY

In the sequence of figures which follow, giving path separation (figures 7a-d) and mean search time (figures 8a-d) as functions of location probability, lateral range function, swath width, and RMS deviation, only those values are shown which satisfy the acceptability criterion

$$\frac{\|\phi_d - \phi\|}{\phi} \leq 0.05$$

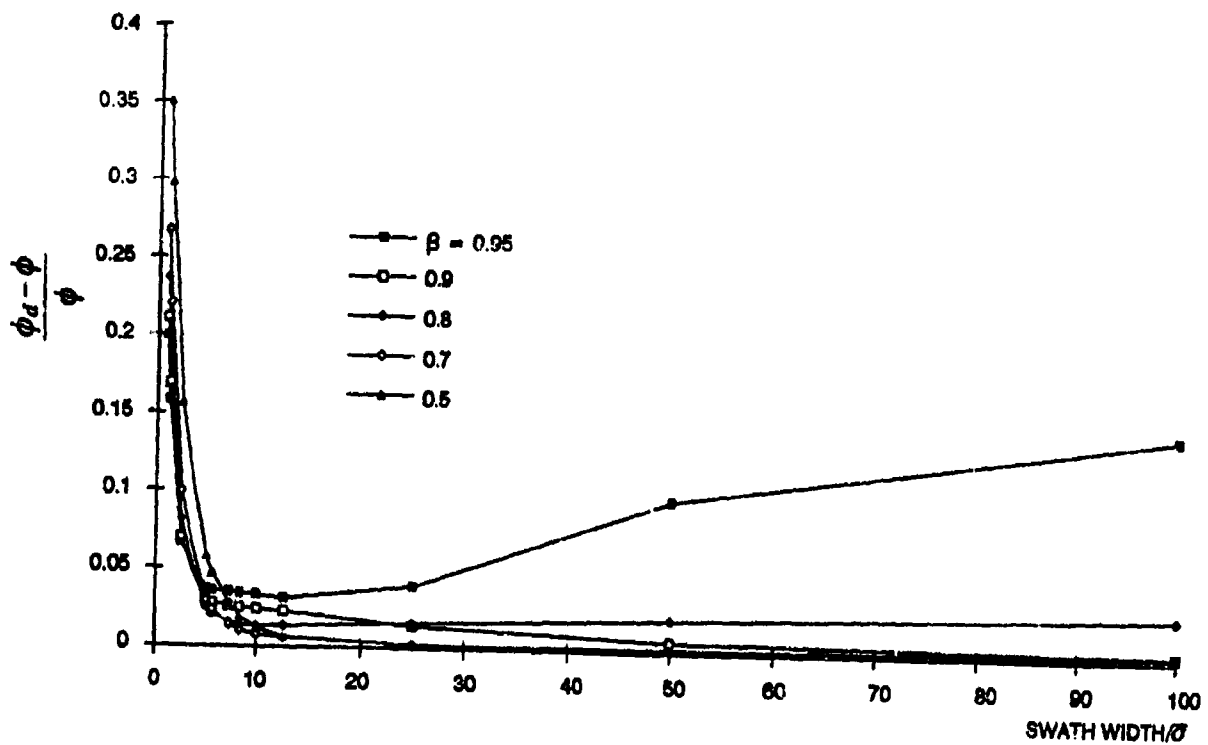
The measure of mean time to detection, which is given in figures 8a-d, namely T' , is derived from the mean search time discussed above:

$$T_s = \sum_{n=1}^{\infty} \phi(1 - \phi)^{n-1} T_s(n) = \sum_{n=1}^{\infty} \phi(1 - \phi)^{n-1} \frac{nHL}{dv} = \frac{HL}{dv\phi}$$

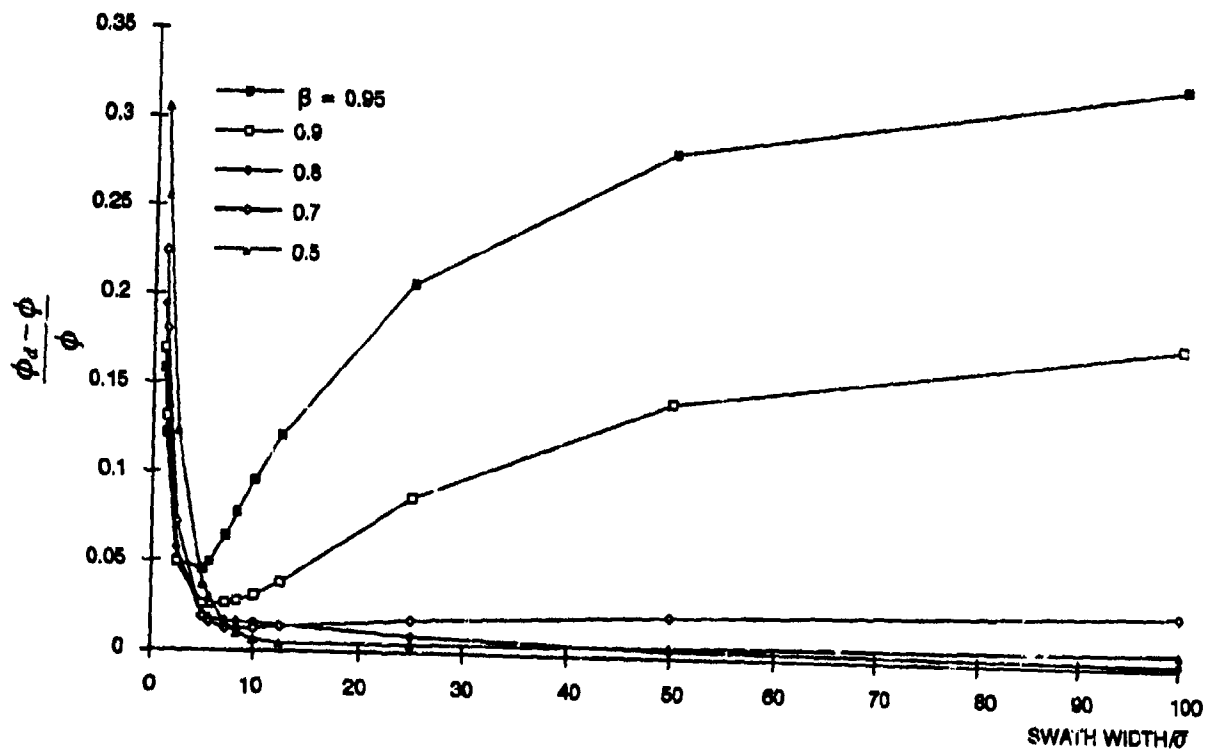
T' is a scale version of T_s :

$$T = \phi \frac{v\sigma}{LH\left(\frac{L}{L}\right)} T_s = \frac{\sigma}{d}$$

Note how this scaled estimate of mean search time is directly proportional to the path length multiplier $a = \frac{L}{L}$ and the value of T' actually plotted is $\frac{\phi d}{\phi} T'$. More complete studies of mean time to detection for the basic Reber model of the Advanced Unmanned Search System are included in Grace, 1986, 1991.



(a) For location probability = 0.90.



(b) For location probability = 0.30.

(Contd)

Figure 6. Approximation error as a function of swath width.

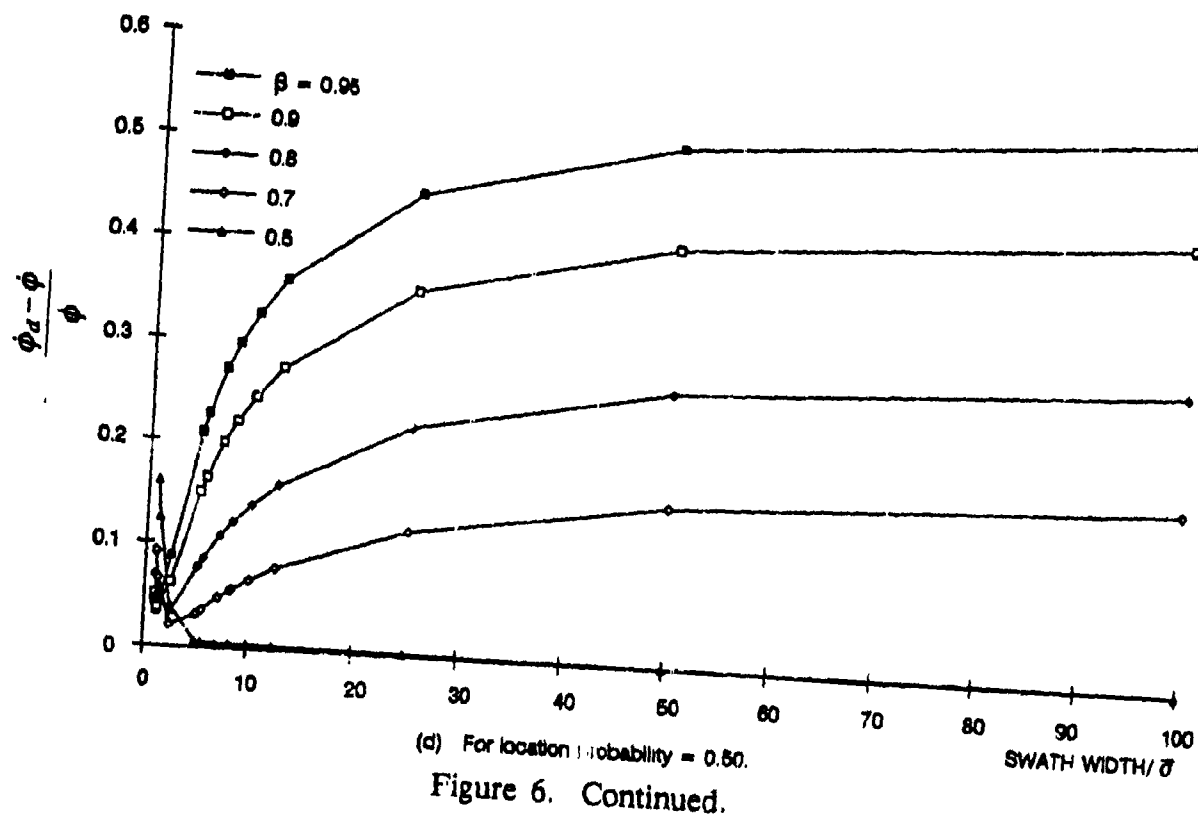
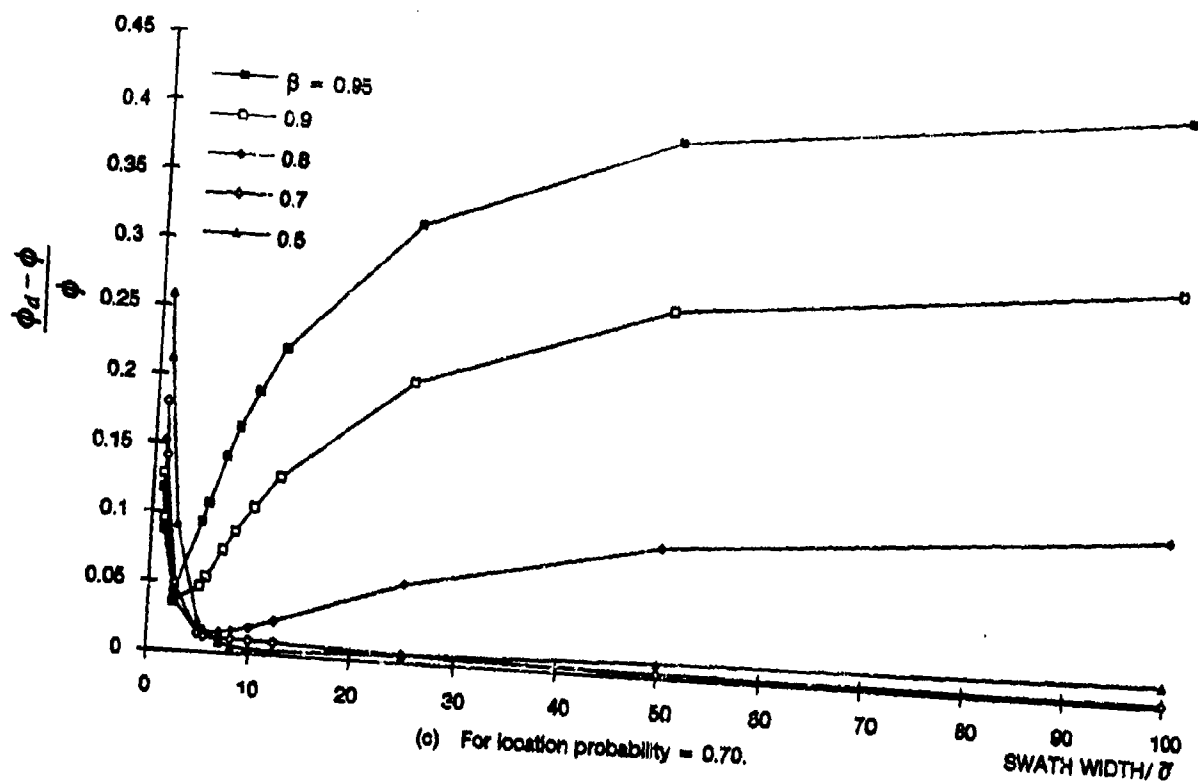
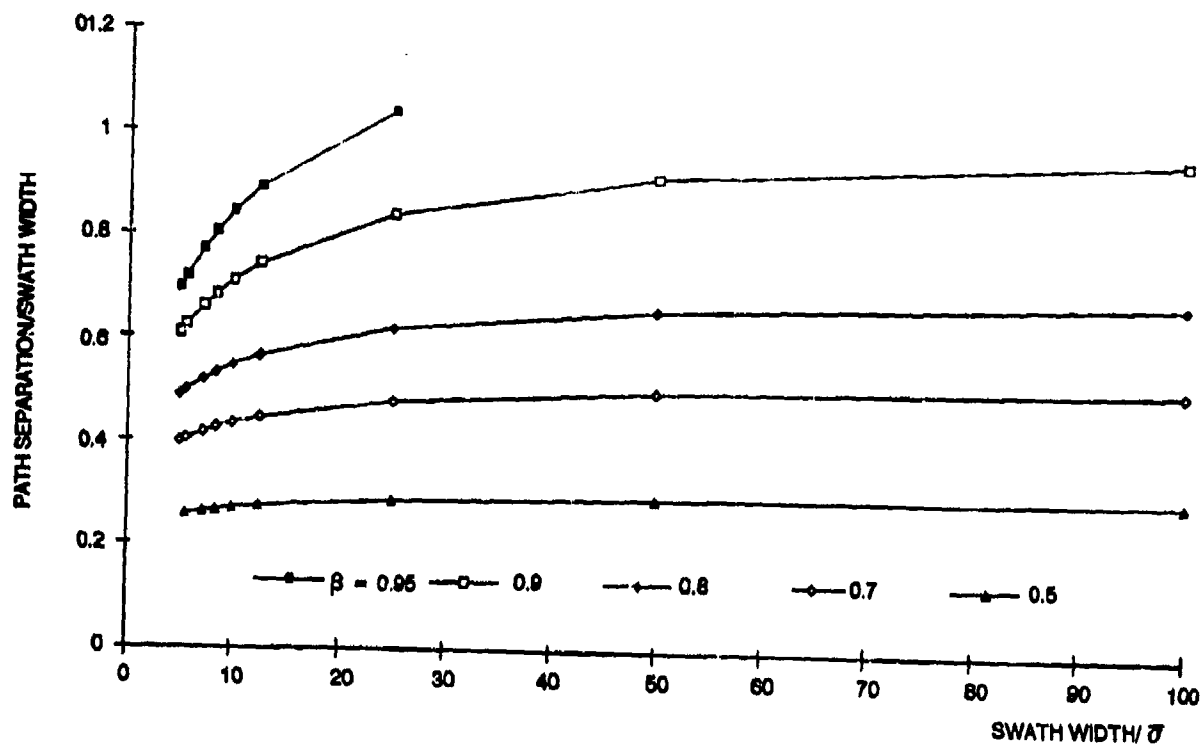
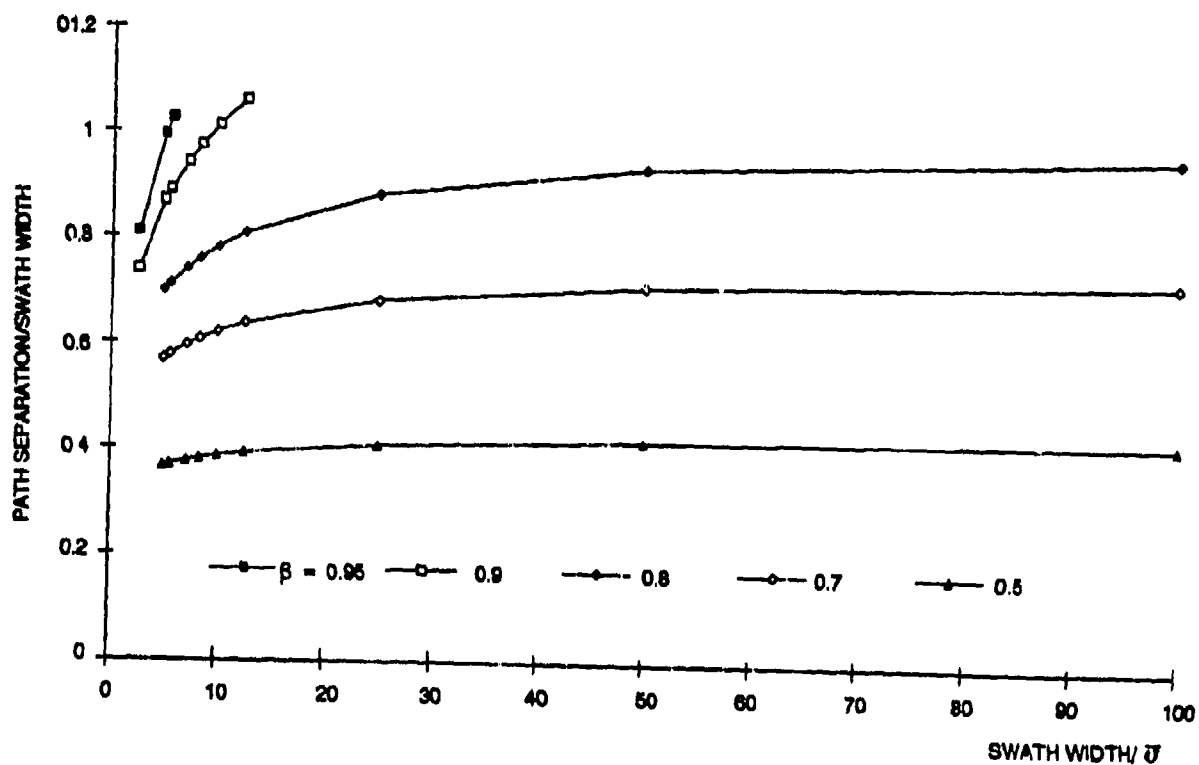


Figure 6. Continued.



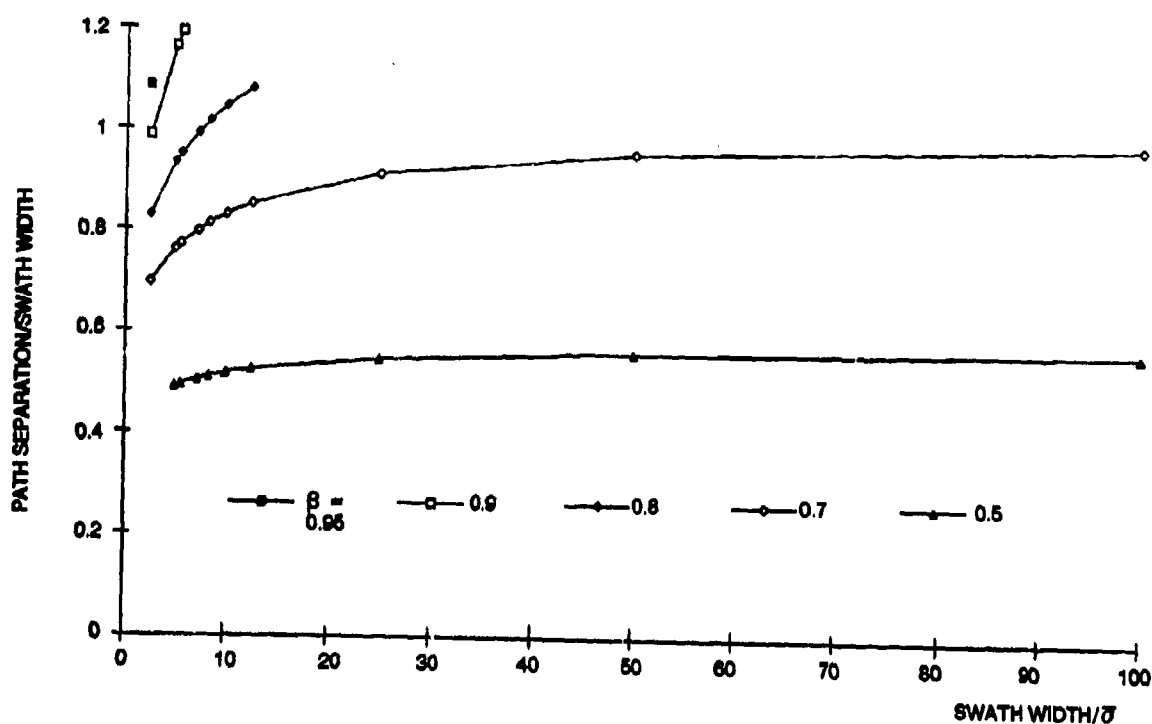
(a) For location probability = 0.90.



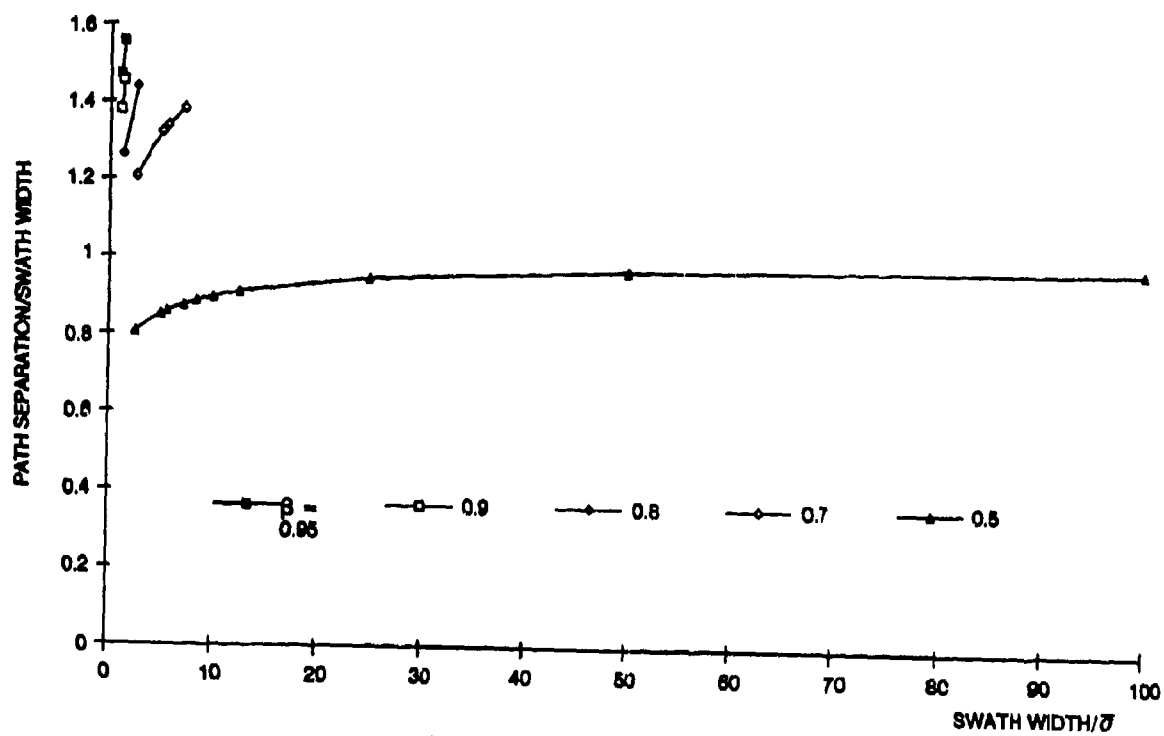
(b) For location probability = 0.80.

(Contd)

Figure 7. Path separation as a function of swath width.

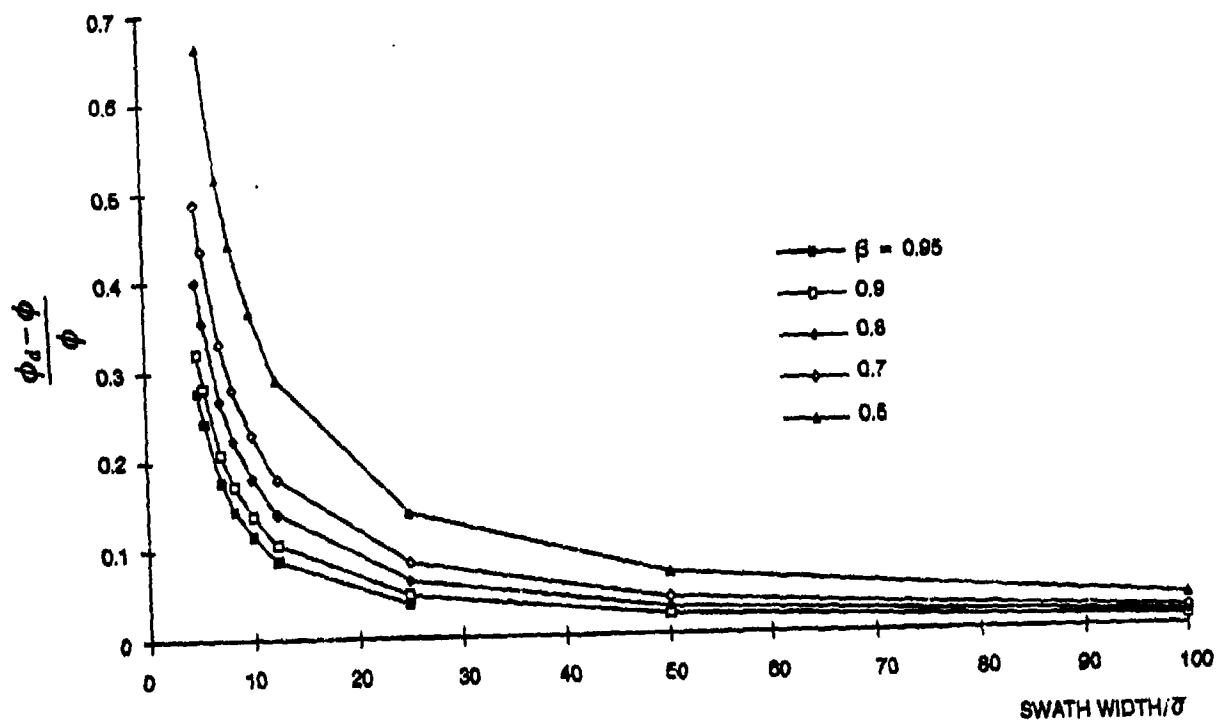


(c) For location probability = 0.70.

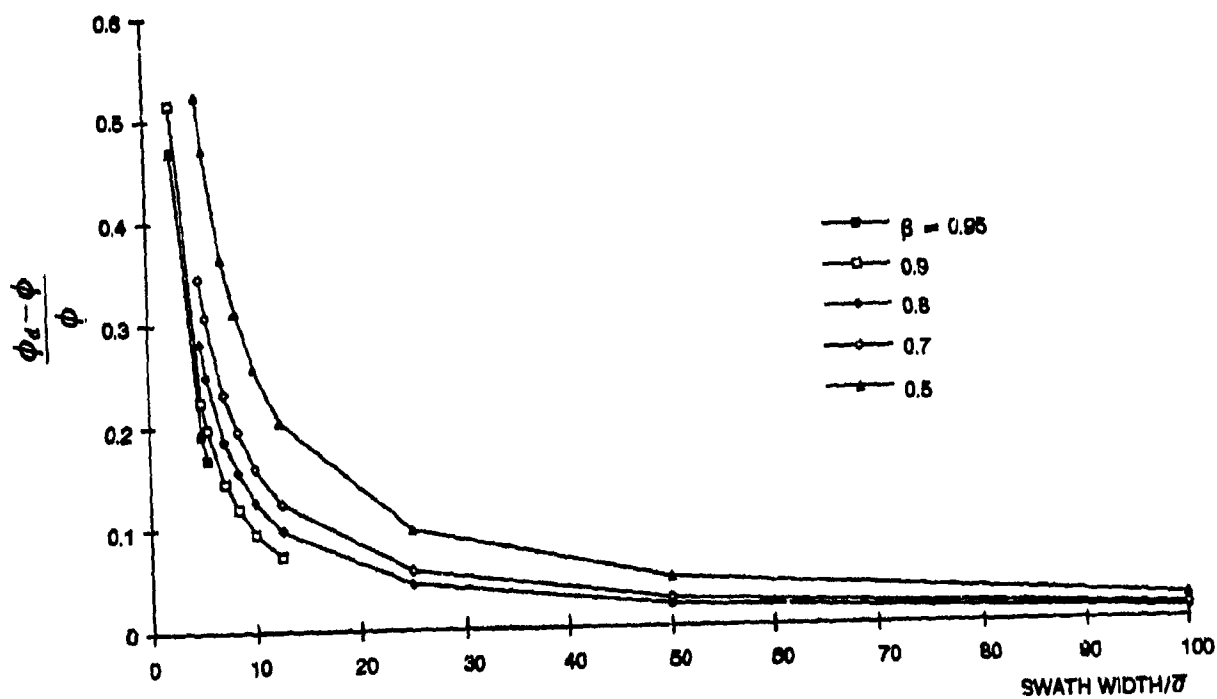


(d) For location probability = 0.50.

Figure 7. Continued.



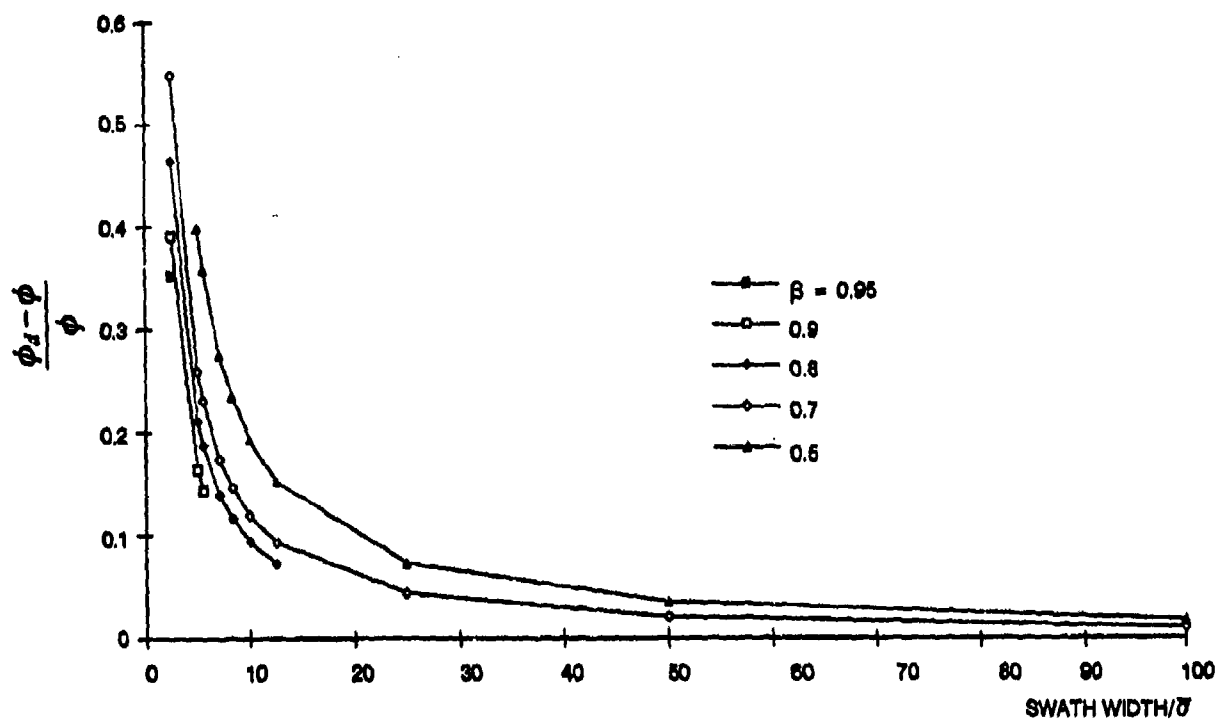
(a) For location probability = 0.90.



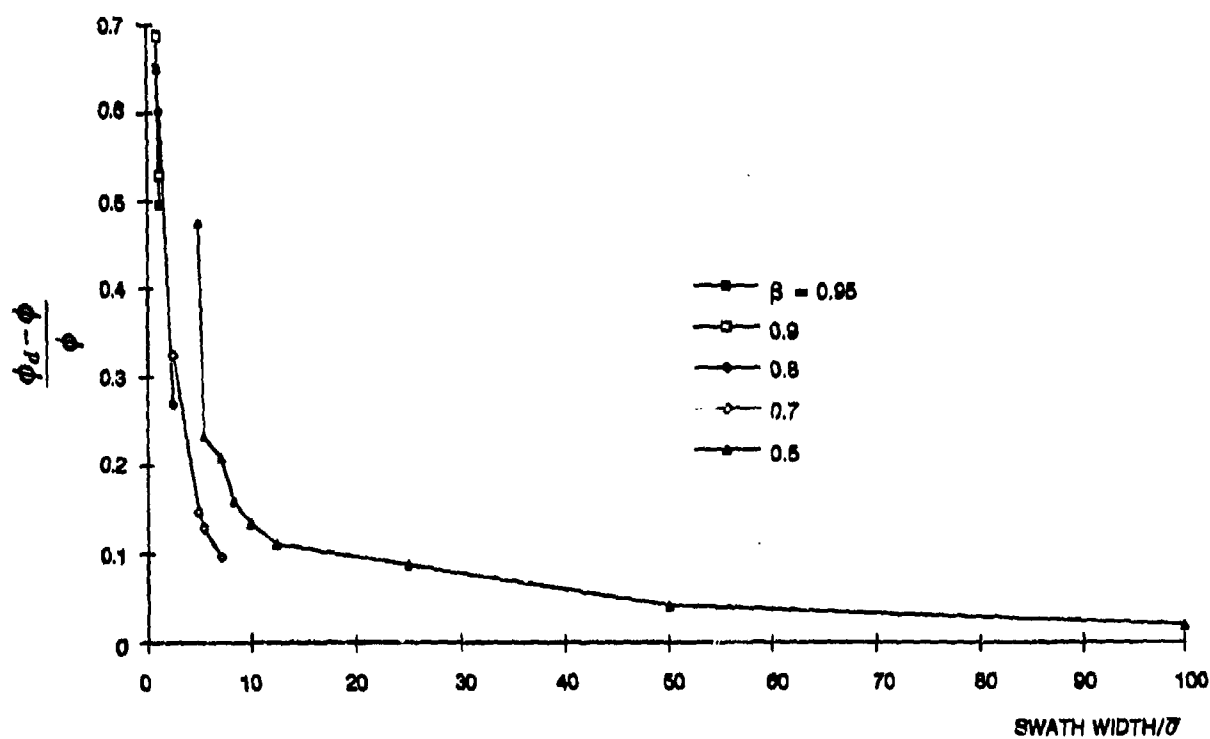
(b) For location probability = 0.80.

(Contd)

Figure 8. Mean search time as a function of swath width.



(c) For location probability = 0.70.



(d) For location probability = 0.60.

Figure 8. Continued.

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